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Flow speed estimated by inverse modeling of sandy sediment deposited by the 29 September 2009 tsunami near Satitoa, east Upolu, Samoa

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Abstract

Sandy deposits from the 29 September 2009 tsunami on the east coast of Upolu, Samoa were investigated to document their characteristics and used to apply an inverse sediment transport model to estimate tsunami flow speed. Sandy deposits 6 to 15 cm thick formed from ~25 to ~250 m inland. Sedimentary layers in the deposits, that are defined by vertical grain size variation and contacts, are interpreted to have formed during onshore runup of two waves. Deposits at 3 locations (100, 170, and 240 m inland) contained two layers that are predominately normally graded (~80%), but contained massive sections (~15%) and inversely graded sections (~5%) at their bases. About 75% of the total thickness of normally graded intervals exhibits a signature of sediment falling out of suspension at their top. This type of grading, termed suspension grading here, was first recognized in turbidity current deposits and is characterized by the entire distribution shifting finer upwards in a layer as high-settling velocity, coarser material deposits first and low-settling velocity finer material deposits last. The Jaffe and Gelfenbaum (2007) inverse sediment transport model was applied to intervals within layers that exhibited suspension grading to estimate tsunami flow speed and was able to reproduce the general trends of the observed suspension grading. A key unknown input in the modeling is the bottom roughness. For a bottom roughness parameterization using a Manning’s n of 0.03 (equivalent to a z0 ~0.006 m for the observed flow depths of 2–3 m) flow speeds calculated for the 2 layers at the 3 locations were 3.8, 3.6, and 3.7 m/s (bottom layer/earlier wave) and 4.4, 4.4, and 4.1 m/s (top layer/later wave) at 100, 170, and 240 m inland, respectively. These estimates are consistent with the ~3–8 m/s tsunami flow speed from boulder transport calculations and result in Froude numbers of ~0.7–1.0 when maximum measured flow depths are used. Because the inverse model assumes the deposit was formed by sediment falling out of suspension flow speed estimates. Including intervals deposited by either bedload or suspended load transport convergences result in higher, and sometimes unrealistic, tsunami flow speed estimates.

Keywords: tsunami sediment transport Samoa inverse model suspension grading

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1. Introduction

On September 29, 2009, a magnitude-8.1 submarine earthquake occurred at 6:48 a.m. Samoa Standard Time approximately 190 km (120 mi) southwest of Samoa. The initial rupture occurred near the north end of a 3000-km-long segment of the Pacific/Australia plate boundary that trends north-northeast and is marked by the Tonga trench (United States Geological Survey National Earthquake Information Center Web Site) (Fig. 1). The earthquake occurred as a normal-fault rupture within the outer rise of the subducting Pacific plate, where the plate bends downward toward the Earth’s mantle (Okal et al., 2010). This earthquake triggered two major intraplate aftershocks at the northern end of the Tonga subduction zone less than 2 min after the outer-rise earthquake (Beavan et al., 2010; Lay et al., 2010). The combination of these earthquakes caused seafloor deformation that produced a tsunami that resulted in 191 deaths and widespread damage in Samoa, American Samoa, and Tonga (Federal Emergency Management Agency, 2009; United Nations Office for the Coordination of Humanitarian Affairs, 2009; National Oceanic and Atmospheric Administration National Geophysical Data Center, 2011). The vast majority of the deaths (149) occurred on the islands of Samoa.

On the island of Upolu, Samoa, the tsunami reached elevations greater than 14 m above sea level and flooded regions more than 400 m inland (Dominey-Howes and Thaman, 2009; Okal et al., 2010; Fritz et al., 2011-this issue). While this paper focuses on the tsunami characteristics at Satitoa in the Aleipata District of eastern Upolu, Samoa (Fig. 1). The deposit was mainly sand, but also included mud caps, debris piles (vegetation, refuse, etc.), and gravel (pebbles, cobbles, and boulders) buried within the deposit and at the surface. This paper focuses on the sandy portion of the deposit and asks the question, “What information about the tsunami can be extracted from the deposit?” In addition to qualitative information about the tsunami (e.g., number of large waves), an inverse tsunami sediment transport model (Jaffe and Gelfenbaum, 2007; Appendix A) is applied to estimate tsunami flow speeds from the Satitoa deposits.

2. Suspension grading in tsunami deposits

Normal grading is often reported as a feature of tsunami deposits (Bourgeois, 2009 and references therein). However, normal grading is associated with other phenomena and can be created by bedload as well as suspension processes (Bridge, 2003; Bridge and Demicco, 2008). The scale (lamina, layer, sequence) and details of grading can, and should, be used to further constrain whether a deposit can be identified as forming from a tsunami, and if so, the process of formation (bedload vs. suspend load deposition).

A special case of normal grading, here termed suspension grading, is observed in the tops of layers in tsunami deposits (e.g., Jaffe and Gelfenbaum, 2007; Higman and Bourgeois, 2008) where the entire distribution shifts to finer sizes moving upward in the deposit. This shift is a signature of sediment falling out of suspension and is the predicted grading for clearing of a water column charged with suspended sediment with an exponential decrease in concentration (e.g. produced by an equilibrium between upward diffusion and downward settling of sediment). The shift of the distribution to finer sizes occurs because of the timing of when larger and smaller grains are deposited. Grains with higher settling velocities (larger particles for a given density and shape) deposit first and are absent in the water column during the later stages of deposition. The grains with lower settling velocities take longer to reach the bed and are absent from the bottom of the deposit and present in the top of it. The thickness of the
suspension-graded interval is limited by the amount of sediment in suspension and, for tsunami deposits, is typically 10 cm or thinner.

Fig. 2 illustrates the differences between suspension grading and normal grading. By definition, for a portion of the deposit to be normally graded the grain size must decrease upwards (Nichols, 1999). Normally-graded deposits can contain multi-modal intervals, be either better or more poorly sorted upwards, and can have varying skewness values in different intervals. In addition to an upward decrease in grain size, the entire grain size distribution becomes finer upwards in suspension-graded deposits. Suspension-graded deposits, which are a subset of normal-graded deposits, typically contain unimodal intervals, are better sorted upward, and are more positively skewed at their tops.

Suspension grading was recognized in laboratory experiments investigating deposition in turbidity currents (Kuenen and Menard, 1952; Middleton, 1967). Suspension grading (also known as distribution grading) was created when a deposit formed by sediment settling out of suspension in a low concentration (~20% by volume) flow. Middleton (1967) observed rapid deposition of sediment as the velocity of the current rapidly declined, following the passage of the main 'wave' of the current; this process is not too dissimilar from what would occur as a tsunami wave passed.

3. Methods

3.1. Field

Measurements of land elevation, tsunami deposit thickness, extent and other characteristics, and water levels were made approximately three weeks after the event impacted Satitoa. Four physical tsunami parameters were measured at Satitoa: (1) inundation distance, (2) runup elevation, (3) flow depths, and (4) flow direction.

A combination of handheld GPS, laser range finder and real-time kinematic (RTK) GPS were used for surveying the physical tsunami parameters and the sediment sampling locations. The RTK-GPS was also used for obtaining accurate elevations for profiles and topographic maps around the geological sampling localities. Within each RTK-GPS survey the horizontal and vertical accuracy of each point relative to another is less than 0.02 m. The elevation accuracy relative to mean sea level depends upon the quality of the calibration points. Geodetic benchmarks with elevations known relative to mean sea level were used for calibration for the surveys at Satitoa. Surveys were tied into a geodetic benchmark at Lalomanu. For RTK-GPS measurements, we estimate the uncertainty relative to mean sea level to be ±0.2 m.

Flow depth and direction were documented wherever there was reliable field evidence. Flow depth (i.e. wave height above the local ground elevation) was measured with a laser range finder using field evidence such as scratch marks on tree trunks, removed bark, broken branches, and debris and rubbish trapped in branches. Where the flow depth measurements were coincident with, or close to, an elevation
point measured using the RTK-GPS, the flow heights (elevation above sea level) could be calculated. Flow direction was measured using a compass (either magnetic or GPS) from the alignment of bent vegetation (palm trees or coconut trees), collapsed building structures (fale which is a traditional building, pillars or water pipes), fence posts anchored on the seaside walls of some partially destroyed building, and material such as metal roofing wrapped around tree trunks.

The general physical characteristics and trends of tsunami deposition were documented as follows. For the sandy deposits, we dug 10 trenches through the tsunami deposits into the underlying sediments at 25-m intervals along a shore-normal transect (Fig. 3). After cleaning the faces of the trench, we photographed and described deposit characteristics including vertical and horizontal variations in sediment grain size and color, layering, presence or absence of sedimentary structures, and nature of contacts. We also photographed cleaned trench faces to further document the deposit. Sediment samples were taken at 1-cm vertical intervals through the tsunami deposit and into the top of the underlying soil. For the boulder deposits, we documented boulder size and orientation using a tape measure and compass (see Etienne et al., 2004–2011-this issue, and Richmond et al., 2011-this issue, for details on boulders at and near the study site).

3.2. Laboratory

The grain size of the fine sediment was analyzed using three methods: settling tubes, dry sieving, and laser diffraction, all to the 1/4 phi interval. Samples were wet sieved at 4 phi (0.063 mm) to separate silt and clay from the sand and larger material. The sand portion of a sample was run through a long (2 m) settling tube. Measured settling velocities were converted to sizes of equivalent quartz spheres in 1/4-phi intervals for figures in this paper and calculation of statistics (Carver, 1971). The measured settling velocities are used as inputs in the Jaffe and Gelfenbaum (2007) inverse model. Particles larger than the settling tubes can successfully process, typically those larger than −0.5 phi (−0.7 mm), were dry sieved. Silt and clay was run through a laser-diffraction particle analyzer. The results from the three grain size methods are merged into a final complete grain size distribution ranging for −2 to 14 phi.

4. Inverse modeling of tsunami flow speed from the deposit

Flow speed during the 2009 South Pacific tsunami is calculated from the Satitoa tsunami deposits using the approach of Jaffe and Gelfenbaum (2007). This is an inverse approach that determines the flow speed necessary to suspend the sediment deposited in the suspension-graded intervals of the deposit. The model calculates multi-class sediment suspension and assumes local equilibrium between turbulent suspension and settling. Standard sediment transport formulae are used to determine the concentration profile for each size class. A Rouse-type expression (formulation in Appendix A and Jaffe and Gelfenbaum, 2007) is used to calculate sediment concentration in the water column that results from upward diffusion in balance with downward settling of grains. This results in an exponential decrease in suspended sediment concentration with distance from the bed with a greater decrease for higher settling velocities (sediment tends to be lower because it settles faster) and lower flow speeds that generate less turbulence for the same bottom roughness (sediment tends to be lower because the energy to mix it up into the water column is less).

The model iteratively adjusts the input sediment source distribution and shear velocity ($U_s$, a parameterization of sediment pick-up and turbulent mixing intensity) to match the size distribution and amount of sediment in suspension to that of a suspension-graded interval of a layer of the tsunami deposit. This is done for each size class, $i$, as

$$
\frac{h}{\int_0^{z_{\text{top}}} C_i(z)dz} = \frac{z_{\text{top}}}{\int_{z_{\text{bot}}}^{z_{\text{top}}} C_i(z)dz}
$$

where $h$ is the flow depth, $C_i(z)$ is the sediment volume concentration of the size class $i$ at elevation $z$ above the bed, $z_{\text{bot}}$ is the bottom of a suspension-graded interval of the tsunami deposit and $z_{\text{top}}$ is the top of a suspension-graded interval of the tsunami deposit.

A key assumption of the model is that a suspension-graded interval of the deposit formed entirely from sediment falling out of suspension as the flow speed decreased—not from bedload or from sediment flux convergences such as would occur where the flow was decelerating in the direction of travel. If portions of the interval were deposited by bedload or sediment flux convergences, the model
overestimates tsunami flow speed (Jaffe and Gelfenbaum, 2007; Spiske et al., 2010).

After determining the shear velocity needed to produce a suspension graded interval of a deposit, the flow speed profile, \( U(z) \), is calculated by

\[
U(z) = \frac{z}{K(z)} \int_{z_0}^{z} U^2_0 \frac{dz}{K(z)}
\]

where \( z_0 \) is the bottom roughness and \( K(z) \) is the eddy viscosity (see Appendix A for eddy viscosity formulation).

The model is applied here in two ways. First, the vertical variation in grain size distributions of a suspension-graded portion of the tsunami deposit is reconstructed and compared to the actual distributions of the tsunami deposit sampled in the field. The reconstruction follows the methods of Jaffe and Gelfenbaum (2007) and as applied by Witter et al. (2008) to Cascadia paleotsunami deposits at Cannon Beach along the coast of Oregon. The reconstruction is simple—sediment in suspension is allowed to settle and the amount settling in each size class is tracked as the deposit accretes according to

\[
K(z) = k \frac{w_{si}}{U(z)} \left( \frac{z}{w_{si}} \right)^{-1.2} (\frac{z}{w_{si}})^{0.2} + \frac{z}{w_{si}}
\]

where \( C_{si}(t) \) is the volume concentration of particles in size class \( i \) depositing on the bed surface at time \( t \) and \( w_{si} \) is the settling velocity of size class \( i \) particles. The right side of Eq. (3) is the flux of particles depositing on the bed at time \( t \) and is applied discretely for each size class only at elevations where \( z = t/w_{si} \). For example, at \( t = 10 \) s, the flux depositing on the bed that would be applied for pre-settling elevations of 20, 10, and 5 s are for particles with settling velocities of 2, 1, and 0.5 cm/s, respectively. Rather than calculate what elevations are appropriate for a given time, it is more efficient to reconstruct the deposit by calculating the times of deposition for all combinations of height above the bed and settling velocities while tracking concentrations for each combination. The resulting times are ordered from shortest to longest and the bed accreted with time according to the associated concentrations of each size classes.

If the grading of the reconstructed deposit matches that of the observed deposit reasonably well, then the assumption that the sediment was deposited from suspension is supported and the application of the model is likely appropriate (Jaffe and Gelfenbaum, 2007). The tsunami flow speed calculated by Eq. (2), which uses bottom roughness and eddy viscosity to relate the shear velocity to flow speed, is more likely to be a good estimate of the actual flow speed during the tsunami than if the grading of the reconstructed and observed deposits do not match well.

5. Physical setting at Satitoa

The island of Upolu is a high-relief volcanic island with numerous embayments and an irregular coastline. Coastal characteristics of Upolu include: a) a steep, often cliffed shoreline with little or no reef development, b) fringing reef fronting a narrow coastal plain supporting beaches, barrier spits, and coastal wetlands with associated streams, and c) wide fringing reef transitioning to a shallow barrier reef (Richmond, 1992, 1995). The Satitoa study area lies on the eastern windward coast of Upolu in an area bordered by a wide shallow fringing reef and several small offshore islands creating a complex coastal physiography (Figs. 4 and 5). The sandy coastal plain approaches 300 m in width with the seaward margin marked by chronic erosion as evidenced by near-continuous seawalls along the shoreline protecting the coastal road. The landward margin of the coastal plain abuts against a gradually ascending alluvial slope. Wetlands are common along the contact between the sandy coastal plain and the gently sloping uplands. The coastal plain is low-relief and gently undulating without well-defined beach ridges, possibly the result of gardening activities due to long-term human occupation. Several villages and their associated subsistence gardens are situated along the coastal plain.

6. Tsunami characteristics at Satitoa

6.1. Eyewitness accounts

In southeastern and eastern Samoa the majority of eyewitnesses described “two, sometimes three, incoming waves, arriving only a few minutes apart and with no complete withdrawal between.” Eyewitnesses near Satitoa recall two tsunami waves, with the second wave being the strongest (Dominey-Howes and Thaman, 2009).

Fig. 4. Oblique aerial photograph of the Satitoa study area several days after the 29 September 2009 tsunami (photograph courtesy of New Zealand Air Force). Note that water is ponded in lows and the location of the transect line.
6.2. Field observations

Fig. 6 summarizes the tsunami data collected from October 16 to 19, 2010, near the village of Satitoa in the area of the shore-normal transect. In addition to approximately 20 tsunami flow depths and 15 flow directions measured within about 100 m of the shore-normal transect, trench locations, and topographic survey points are shown. The flow direction indicators were oriented either roughly shore-parallel towards the northeast or obliquely onshore towards the north, indicating that the flow did not come in perpendicular to shore. Tsunami flow depth near the transect decreases, in general, landward from a high of ~4 m near the shoreline to 2 m about 200 m inland (Fig. 6). There is also a trend of decreasing flow depth near the shoreline from southwest to northeast (~5 to 2 m), the primary observed flow direction (Fig. 6). The variation in flow depth near Satitoa is a simple one with a monotonic (within measurement error) landward decrease, similar to observations in other tsunamis (e.g., Papua New Guinea 1998; Gelfenbaum and Jaffe, 2003). The complexity in tsunami flow direction, however, is expected given the variability in offshore bathymetry, and is supported by flow direction indicators and eyewitness reports.

There was no evidence of strong return flow, but there was evidence of water collecting in topographic lows and remaining after the tsunami. Water was still ponded in topographic lows several days after the tsunami as shown in oblique aerial photographs taken by the New Zealand Air Force (Fig. 3).
At Satitoa, erosion was prominent at the shoreline, along the shore-parallel coast road, and approximately the first 25 m of the coastal plain. Sediments entrained in the tsunami were deposited inland of the erosion zone and included sand sheets, gravel fields and isolated boulders, mud caps, and accumulations of organic and man-made debris. This study focuses on the sandy deposits observed and sampled in shallow trenches along a shore-normal profile.

Sand deposits ranged from very thin patches (<1 cm) to broad sand sheets up to 10 s of centimeters thick. Along the shore-normal profile near Satitoa the average thickness of sand exposed in trenches ranged from 6 cm to nearly 12 cm (Fig. 7). Localized thick sand accumulations were common in the lee of structures, such as low walls, and in topographic depressions. The overall vertical sequence in the sand sheets consist of an erosional base of typically brownish soil overlain by a massive appearing and/or fining upward coarse white coral sand, an upper sand unit often with more than one fining-upward fine lamination, capped by a mud unit a few millimeters to nearly 5 cm thick. The thicker sand deposits had multiple and often complex internal laminations. Based upon color and composition, the sand appeared to have been derived from multiple sources based on color and composition. Reef sediment included fresh coral and articulated Halimeda fragments. Sediment eroded from the beach, backbeach, and seaward edge of the coastal plain (brown sandy soil) also contributed to the deposit.

Our grain size analyses and modeling concentrated on deposits from three trenches (T13 at 100 m inland; T16 at 170 m inland; and T19 at 240 m). These sites were chosen because they had appropriate field sampling and the inland variation in calculated tsunami flow speed could be explored in the ~150 m of coast that the samples spanned. Vertical grading of the deposits is used to interpret tsunami wave signatures and to select suspension-graded intervals for input to the inverse model.

7.1. Deposit grading

Tsunami deposits from the 3 trenches were predominately normally graded (~80% of the deposit). Deposits also contained massive (~15%) and inversely graded sections (~5%). About 75% of the total thickness of the normally-graded intervals exhibit suspension grading.

The vertical variation in grain size at trench 13 is shown in Fig. 8 with two normally graded intervals (0–4 cm and 5–8 cm). Both intervals are suspension graded. Figs. 8 and 9 show a shift of the distribution to finer sizes upwards in the layer at 5 to 8 cm depth. The sample at a depth of 4–5 cm shows a bimodal distribution, which could be caused by the sample capturing the top of the lower layer and

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**Fig. 7.** Topography, tsunami wave height, and deposit thickness on the shore-normal transect near Satitoa. Open circles indicate trenches for which detailed analyses and inverse sediment transport modeling were performed.

**Fig. 8.** Vertical variation in grain size distributions and photograph of cleaned face at T13, which is 100 m inland. Sediment from 0 to 4 cm and from 5 to 8 cm below the surface are suspension graded. Lines are drawn interpreting the elevations where distributions and photographs match. It is not expected that there be an exact match because the deposit was sampled about ~0.5–1 m from where the photograph was taken and there is small-scale variability in the deposit. Another reason for a mismatch is that the photograph is of a low angle face that expands the scale of vertical variation. If the oblique cut was not flat, then there would be distortion in the photograph.
the bottom of the upper layer. In both layers, shells and shell fragments were more abundant towards the base.

The deposit at trench 16 also has two layers (from 0 to 3 cm and from 3 to 7 cm) both of which contain normally graded intervals (Fig. 10). Both intervals are suspension graded (0–3 cm and 3–6 cm), but grading is not as evident as in trench 13. The sample for the interval from 6 to 7 cm was lost in transport and, to represent the correct overall thickness, the distribution for sample from 5 to 6 cm is duplicated for this interval. Shells and shell fragments were not as abundant at T16 as at T13.

At trench 19, which is within ~10 m of the limit of tsunami inundation, grain size distributions also define two layers, although neither is entirely normally graded (Fig. 11). The lower layer (4–7 cm depth) is uniform in the bottom 2 cm and then fines in the top centimeter. It could be viewed as a massive sublayer overlain by a normally-graded one. Massive lower intervals of layers have been observed in other tsunami deposits (Jaffe et al., 2006; Higman and Bourgeois, 2008). The top of the lower layer appears to be suspension graded. The upper layer, from 0 to 4 cm depth, also has suspension grading at the top (0–2 cm) but is more massive near its base. The trench photo shows alternating lighter and darker bands; however, these do not correlate with grain size changes from the laboratory analyses.

8. Inverse modeling

Inputs to the inverse model are thickness of the suspension-graded deposit interval, its bulk grain size distribution, and the measured tsunami flow depth. Six vertical intervals, one in each layer at the three trench locations, with suspension grading were selected for modeling (Table 1).

The range of grain sizes input for the bulk distribution in the inverse model ranged from 0.5 to 14 phi. Particles larger than 0.5 phi, whose size necessitated analysis by sieving, were almost entirely platy shell fragments and would be assumed to be siliciclastic grains in the standard algorithms that convert grain size to settling velocity. Shells and shell fragments settle more slowly than siliciclastic grains because of their shape and density (Woodruff et al., 2008) and treatment as siliciclastic grains would result in the model overestimating tsunami

---

Fig. 9. Comparison of observed and modeled grain size distribution in deposits from T13 from 5 to 8 cm below the surface. Both the observed and modeled distributions exhibit suspension grading, a signature of formation for sediment falling out of suspension. Note that the model, which allows suspended sediment fall to the bed out of equilibrium concentration profiles is not exact, but reproduces the general behavior of the observed grading.

Fig. 10. Vertical variation in grain size distributions and photograph of cleaned face at T16, which is 170 m inland. Sediment from 0 to 3 cm and from 3 to 6 cm below the surface is suspension graded. Lines are drawn interpreting the elevations where distributions and photographs match. An exact match between the photograph and the plot is not expected because the deposit was sampled about ~0.5–1 m to the side from where the photograph was taken and does not account for small-scale variability in the deposit. The sample for the interval from 6 to 7 cm was lost in transport and, to represent the correct overall thickness, the distribution for sample from 5 to 6 cm is duplicated for this interval.
Fig. 11. Vertical variation in grain size distributions and photograph of cleaned face at T19, which is 240 m inland and near (within ~10 m) of a hill and the limit of inundation. Suspension-graded intervals are present 0–2 cm and 4–6 cm below the surface. Lines are drawn interpreting where the plots and photographs match. It is not expected that there be an exact match because the deposit was sampled about ~0.5 m from where the photograph was taken and there is small-scale variability in the deposit. The trench photo shows alternating lighter and darker bands; however, these do not correlate with grain size changes from the laboratory analyses.
laboratory experiments for high concentration mixtures (Amy et al., 2006). This would account for less fine sediment being present in the water column and depositing in the upper part of the interval. The lack of coarser particles in the modeled distributions for 5–6 cm is more problematic. This may indicate a separate depositional process (bedload or sediment transport convergence) that is not accounted for in the model. It is unlikely however that the exclusion of shell material, which was typically less than 5% by weight, significantly affected the model results. The take-home point to this comparison is that, although the observed and modeled distributions are similar, they differ and caution should therefore be used in the calculated tsunami flow speed. Results for comparisons carried out for other intervals and other trenches were similar.

Modeling results are shown in Fig. 13 and presented in Table 1. Calculated tsunami flow speeds for each layer were similar for all trenches. Tsunami flow speeds were slightly greater for the lower layer than the upper one. For a Manning’s n of 0.03, which corresponds to a z0 bottom roughness of ~0.006 m, tsunami flow speeds calculated were similar for each layer (upper: within 7%, lower: within 6%). Calculated flow speeds for the lower layer of the deposit were 4.4, 4.4, and 4.1 m/s at 100, 170, and 240 m inland, respectively. For inverse modeling using sediments of the upper layer, flow speeds were 3.8, 3.6, and 3.7 m/s at 100, 170, and 240 m inland, respectively. Froude numbers for these speeds are in the range of 0.69 to 1.01 assuming that the maximum water depth measured is the appropriate depth (Table 1). The Froude number is defined as:

\[
Fr = \frac{U}{\sqrt{gh}}
\]  

where \( U \) is the flow speed, \( g \) is acceleration due to gravity, and \( h \) is flow depth.

It is interesting that there was not a landward decrease in calculated tsunami flow speed. The lack of a landward decrease was most likely caused by the alongshore flow direction (Fig. 6).

Changing bottom friction has a large effect on calculated tsunami flow speeds (Table 1). Reducing Manning’s n to 0.02, which corresponds to a z0 bottom roughness of ~0.0005 m, increases calculated tsunami flow speeds by 70%. Froude numbers are correspondingly increased to 1.17–1.69. Increasing Manning’s n to 0.04, which corresponds to a z0 bottom roughness of ~0.023 m, decreases calculated tsunami flow speeds by 30%. Froude numbers are correspondingly decreased to 0.50–0.73.

9. Other calculations of tsunami flow speed

9.1. Froude number constraints on flow speed

Another method for estimating tsunami flow speed at Satitoa is to use the measured flow depths and a Froude number. The Froude number is a non-dimensional number that describes whether a flow is supercritical (>1), critical (=1), or subcritical (<1). Rearranging Eq. (4) to solve for flow speed

\[
U = Fr\sqrt{gh}
\]

allows a direct calculation of flow speed if the Froude number and flow depth are known.

The value for the Froude number for tsunamis varies spatially and temporally, but there is guidance on appropriate values to choose. Fritz et al. (2006) calculated Froude numbers ranging from 0.61 to 1.04 from Particle Image Velocimetry analysis of video imagery taken approximately 3 km inland in Banda Aceh during inundation of the 2004 Indian Ocean tsunami. Spiske et al. (2010) state that the Froude

Fig. 13. Inverse sediment transport model performance. a) topography and trench locations, b) tsunami flow depth, c) calculated shear velocity needed to suspend the amount and distribution of sediment in vertical intervals with distribution grading, and d) depth-averaged tsunami flow speed calculated for vertical intervals with distribution grading. Manning’s n values used for the shear velocities calculated by the inverse model in c) and for the depth-averaged flow speeds in d) are 0.02 (green), 0.03 (red), and 0.04 (blue).
number for most parts of a tsunami flow is subcritical, and therefore the Froude number is between 0.7 and 1. However, large Froude numbers are expected in the very tip of the wave front. Matsutomi et al. (2001) calculated Froude numbers of 0.7 to 2.0 for 6 tsunamis using velocities estimated from Bernoulli's principle applied to flow depths measured on the upstream (front) and downstream (rear) walls of houses.

Froude numbers between 0.5 and 1.5 yield tsunami flow speeds in the range of 2 to 10 m/s at Satitoa (Fig. 13). For Froude numbers of 0.7 to 1, tsunami flow speeds would be in the 4 to 7 m/s range.

9.2. Flow speed estimated from boulder transport

Transported boulders in the study area can be used for another estimate of flow speed. Equating the fluid drag force with a resistance force for sliding and solving for flow speed at a reference level above the bed (0.6° up-axis length) gives:

\[ U_{\text{slide}} = \sqrt{\frac{\mu (\rho_0 - \rho_v) V_s}{g B \sqrt{g C_D F_{\text{fr}}}} \left(\frac{\rho_0}{\rho_v}\right)} \]  

(6)

Where \( \mu \) is the friction coefficient (set to 0.7 after Noormets et al., 2004 and Goto et al., 2007), \( \rho_0 \), is particle density (2545 kg/m³), \( \rho_v \) is the fluid density (1025 kg/m³). \( V_s \) is particle volume (assumes rectangular prism), \( g \) is acceleration due to gravity (9.81 m/s²). \( C_D \) is the drag coefficient (set to 1.5, the value used for modeling boulder transport at this site by Etienne et al., 2004–2011; Buckley et al., in press), and \( F_{\text{fr}} \) is the area of the boulder projected normal to the flow (assumes rectangular prism with A-axis [long axis] normal to flow). A boulder begins to move when the flow speed is slightly greater than \( U_{\text{slide}} \). This formulation neglects lift and inertial forces, which, if significant would reduce the flow speed needed to initiate motion of the boulder. Because this formulation is for initiation of motion, the flow speed transporting the boulders at Satitoa could have been higher. Flows speeds (likely minimum estimates) calculated for initiation of motion of boulders and converted to depth-averaged speeds using the same vertical velocity profile as used in the inverse model when Manning’s n is 0.03 are plotted on Fig. 13. Tsunami flow speeds for initiation of motion of boulders range from 3.6 to 8.2 m/s, with most values between 4 and 5 m/s. The majority of the flow speeds calculated from boulders have similar magnitude as flows from the inverse sediment transport model for n = 0.03 (\( U = 4 \) m/s) and for subcritical flow (Froude number from 0.7 to 0.9). For a more detailed analysis of boulder transport during the Samoa tsunami, see Etienne et al. (2004–2011; this issue).

10. Discussion

10.1. Does the model work?

The most obvious question is, “does the model work?” This is not an easy question to answer. No model is perfect. For forward models, one can evaluate whether a model should work based on whether adequate input data is available and if the physics of the model contains the relevant physics of what is being modeled. Similar criteria can be used to assess the suitability of an inverse model, except that the physics may not be as well known because the relative strengths of the forcing (e.g., flow speed) are not “established” until after the model is run. For Satitoa, the parameter we modeled, tsunami flow speed, was not directly measured. The only direct measurements of on-land tsunami flow speed we are aware of were recorded in Banda Aceh, Sumatra during the 2004 tsunami (Fritz et al., 2006). However that setting did not have a natural bed, was 3 km inland, and therefore those measured values are not directly comparable to the values estimated at the Satitoa study area.

We can assess whether a suspension model is appropriate by using a suspension threshold criterion (ratio of shear velocity to grain settling velocity) developed by Bridge (2003). A ratio of the shear velocity (\( U_r \)) to the settling velocity (\( W_s \)) above 2.5 suggests that sediment will predominantly be transported in suspension, assuming that the coefficient of anisotropy reaches a maximum of 0.1 to 0.2 near the bed (Bridge, 2003). The \( U_r/W_s \) for 1 phi sediment in Satitoa was from 2.7 to 4.4. The 0.5 phi sediment meets the suspension threshold criteria for half the inverse runs and is just below it (\( U_r/W_s \) as low as 1.9) for the remainder of them. Additionally, when using shear velocities from flow speeds calculated in boulder transport modeling or from Froude number considerations the suspension threshold criteria is met for Manning’s n greater than about 0.03. The calculated suspension threshold criterion of tsunami deposit sediments in Satitoa is consistent with transport in suspension and supports the use of an inverse model based on suspended sediment transport.

Another way to assess model performance is by consistency of tsunami flow speed predictions and their magnitude relative to other methods. Tsunami flow speeds calculated for the bottom layers for each trench were similar (Fig. 13). Likewise, speeds calculated for the top layers were similar for each trench. Because the trenches were within 140 m of each other and the tsunami approached alongshore or obliquely alongshore similar tsunami flow speeds are a reasonable result. Tsunami flow speeds calculated with the inverse model for Manning’s n = 0.03 and 0.04 were subspectral as is expected (Spiske et al., 2010). However, for Manning’s n = 0.02 all calculated flows were supercritical, with Froude numbers as high as 1.69. Flow speeds calculated for the initiation of boulder transport for the majority of clasts are in the same range (\( -4 \) to \( -5 \) m/s) as those calculated using the inverse model for Manning’s n = 0.03 and 0.04, except for high values near 100 m inland (Fig. 13). In summary, the inverse model appears to perform reasonably well when using a Manning’s n of 0.03 or 0.04.

However, comparisons between observed and modeled distributions reveal that the model may be missing pertinent physics. To quantify model performance it will be necessary to apply the model in a more controlled environment. This could be done in a laboratory (if the natural conditions can be modeled at an appropriate scale) and/or by running the inverse model in tandem with a forward one, such as that used by Apotsos et al. (2009) and vary the forcing and inputs to assess performance. Of course, testing one model against another doesn’t necessarily prove anything—they could give the same results and both are incorrect.

10.2. Suspension grading and application of the inverse model

Whether suspension grading is present should be evaluated before applying the model to estimate tsunami flow speed from the normally graded interval of a tsunami deposit. If suspension grading is not present, the inverse model’s results will be affected because some process other than sediment falling out of suspension (e.g., bedload deposition or spatial convergence in sediment transport) is significantly contributing to deposit formation. Application of the model to vertical intervals of a tsunami deposit where suspension grading is not clear will likely result in an overestimate of tsunami flow speed. This is because sediment assumed to be falling out of suspension as the tsunami rapidly decelerates (a model assumption) is deposited from another process. For example, if the entire layer (from 0 to 4 cm) is used instead of the suspension-graded interval from 0 to 2 cm at T19 as model input, then calculated flow speed increases by 29% (4.77 m/s vs. 3.69 m/s for a Manning’s n of 0.03).

A much-needed area of research is the development of methods for assessing the quality of inverse tsunami sediment transport model

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outputs. Numerical experiments on the effects of including intervals that are not suspension graded in modeling would establish potential errors. Additionally, metrics that define the degree that the observed suspension grading is reproduced and the resulting affect of calculated tsunami flow speed would improve confidence in model results.

10.3. Settling velocity, not particle size, is the pertinent parameter

Spiske et al. (2010) recognized that the best model results were achieved using settling tube data as input. When using settling tube data, the pertinent parameter for the equations for suspension, the settling velocity, is directly measured. Spiske et al. (2010) pointed out that when grain size is measured using standard techniques (sieves and laser diffraction instead of settling tubes) then care must be taken to account for density and shape when converting to settling velocity. Grain size data gathered using these techniques cannot simply be converted to settling velocity with the expectation that the inverse model will perform well. A good example of this is when shells or shell fragments, whose size is measured by sieving, are converted to settling velocity assuming a spherical quartz grain. The inverse model will give unreasonably high tsunami flow speeds because the settling velocities used by the model are too high.

10.4. Importance of bottom friction

The importance of bottom friction cannot be overemphasized. Tsunami flow speeds near Saitoia varied by a factor of more than 2 when Manning's n was changed from 0.02 to 0.04. For this inverse modeling approach to be successful, the bottom friction must be reasonably well known. Knowing this “well enough” is complicated by natural spatial variability and the probability that bottom friction changes throughout most tsunamis in most coastal areas as vegetation is removed by the tsunami and debris piles are formed on the coastal plain. Another factor influencing the effective roughness is that when tsunamis inundate gently-sloping coastal plains, the water does not completely withdraw between waves. This scenario occurred at several locations in Samoa. Furthermore, the presence of ponded water or a water bodies such as lakes or rivers reduces effective roughness. Although bottom friction is not an easy problem to address, it is a worthy research topic if the goal is to improve tsunami hazard assessment using hydrodynamic and sediment transport models.

10.5. Application to paleotsunami deposits

With all the potential pitfalls of the inverse model, one might think it is not ready for application to paleotsunami deposits. However it was applied with some success to deposits from the 1700 Cascadia tsunami at Cannon Beach, Oregon, USA (Witter et al., 2008). The Witter et al. (2008) flow speed estimates were similar, but somewhat higher than hydrodynamic model calculations with the differences possibly being caused by a different treatment of bottom friction. Woodruff et al. (2008), who also modeled deposits, simplified comparison of inverse model results for hurricanes in Puerto Rico by using the relative flow speeds of different storm events. Perhaps a similar approach, where relative tsunami flow speeds were calculated without specifying bottom roughness, could be used for sites where multiple paleotsunami deposits are present.

One encouraging aspect of Witter et al.’s (2008) research was that vertical variations in the observed deposits were reproduced well by the model. However, for this inverse model to become a more effective tool for assessing paleotsunami flow speeds (and heights using assumed Froude numbers) more testing and continued development are needed. Part of this process is applying the model to paleotsunami deposits and evaluating and comparing the results to other methods including, hydrodynamic modeling.

A final step that is needed to improve confidence in the application of the inverse model to paleotsunami deposits is to determine how post-tsunami alteration (taphonomy) affects the model. It is possible that for some (most) deposits the signature of suspension grading could be completely obscured by post-deposition alteration. If so, modeling a portion of the deposit thicker than the suspension-graded interval would result in an overestimate of tsunami flow speed. Counter to this is the possibility that post-depositional erosion occurred, in which case the entire suspension-graded interval would not be modeled. This would lead to an underestimate of the tsunami flow speed that created the deposit. We expect that the post-depositional loss of the sedimentary record by erosion is a greater problem for the uppermost layer when it is not capped (protected) by mud and/or organics. More work needs to be done to develop criteria to determine the degree of post-deposition alteration that is too great to yield accurate results when applying the inverse model.

One advantage to an inverse approach for calculating tsunami flow speed is that small changes in the deposit do not lead to large variations in speed. Because sediment transport is highly nonlinear (transport is a function of flow speed to a power greater than 1, usually 3 or more) small changes in flow speed will have large effects on tsunami deposit characteristics (e.g., thickness). The opposite is true for a model where the sediment transport is calculated from the deposit. Variations in the input (deposit characteristics) result in smaller changes in the calculated tsunami flow speed (function of deposit characteristics to a power less than one). This bodies well for the inverse approach because the effect of variability that is inevitable for data collected in the field data has limited consequences for the output. It would be a much more difficult (perhaps intractable) problem to solve in the forward mode because small changes in flow speed result in large changes in the deposit and uncertainties in input (speed) would be amplified in the output (tsunami deposit characteristics).

11. Summary and conclusions

Deposits from the 29 September 2009 tsunami were investigated to determine what flow characteristics they recorded. Near Saitoia on the east coast of Upolu, deposits were laid down between ~25 and ~250 m inland, landward of a zone of erosion and were from 6 to 15 cm thick. Based on vertical grain size variation and contacts we identified two layers and attribute their formation to uprush from two tsunami waves. Deposits at 3 locations (100, 170, and 240 m inland) were predominately normally graded (~80%), but contained massive (~15%) and inversely graded sections (~5%). About 75% of the total thickness of the normally graded intervals exhibited a textural signature of sediment falling out of suspension, suspension grading, where coarse material is lost up-layer and fine material gained.

In order to estimate tsunami flow speed the Jaffe and Gelfenbaum (2007) inverse sediment transport model was applied to vertical intervals within the deposit that exhibited suspension grading. The upper intervals in all three deposits had similar calculated tsunami flow speeds (3.7 m/s ± 0.1 m/s) and are interpreted as being deposited by the same, later wave. Calculated tsunami flow speeds for the lower intervals were greater (4.4 m/s ± 0.3 m/s) and are interpreted to be deposited by the same, earlier wave. Froude number considerations and boulder transport modeling resulted in similar flow speed estimates.

This approach of interpreting tsunami characteristics from their deposits is promising, but in its infancy. Tsunamis such as the 29 September 2009 event offer an opportunity to improve our ability to interpret tsunami deposits and their relationship to the formative
processes. Additional research is needed that combines hydrodynam-
ic and sediment transport modeling (both inverse and forward) and
detailed analysis of deposits from recent tsunamis to establish
the relationship between tsunami characteristics and the resulting
deposit.

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Appendix A

Equations from Jaffe and Gelfenbaum (2007) model, plus several
new ones, are presented below. The sediment concentration in the
inverse tsunami sediment transport model is given by:

\[ C_i(z) = C_{ri} e^{-\frac{w_i}{\tau_{cri}}} \]  

(A1)

where \( C_{ri}(z) \) is the sediment concentration of size class \( i \) at the
elevation \( z \) above the bed, \( C_{ri} \) is the reference concentration for size
class \( i \), \( W_i \) is the sediment settling velocity, \( \zeta_i \) is the bottom roughness
parameter, and \( K \) is the eddy viscosity (a function of \( U_e \), the shear
velocity and distance above the bed).

Reference concentration is calculated following Madsen et al.
(1993) as

\[ C_{ri} = \frac{y_0 f_i S}{1 + y_0 S} \]  

(A2)

in which \( y_0 \) is the resuspension coefficient, \( C_{ri} \) is bed concentration, \( f_i \)
is the fraction of bed sediment in size class \( i \), and \( S \) is normalized excess shear stress given by

\[ S = \frac{\tau_b - \tau_{cri}}{\tau_{cri}} \quad \text{if} \quad \tau_b > \tau_{cri}, \]  

(A3a)

\[ S = 0 \quad \text{if} \quad \tau_b \leq \tau_{cri}, \]  

(A3b)

where \( \tau_b \) is the bed shear stress, is

\[ \tau_b = \rho_u U_e^2 \]  

(A4)

with \( \rho_u \) the density of water and \( U_e \) the shear velocity, and the critical
shear stress for the initiation of motion is

\[ \tau_{cri} = \rho_u U_{cri}^2 \]  

(A5)

where \( U_{cri} \) is the critical shear velocity for initiation of motion for size
class \( i \).

To calculate reference concentration, we must choose a method for
calculation of \( U_{cri} \) and values for \( y_0 \) and \( C_{ri} \). We calculate \( U_{cri} \) following
Madsen et al. (1993). We use the Hill et al. (1988) value of 1.4 \times 10^{-4}
for \( y_0 \) and 0.65 for \( C_{ri} \) (Smith and McLean, 1977).

To solve Eq. (3), sediment settling velocity, bottom roughness, and
eddy viscosity must be specified. Settling velocity may be calculated
from grain size and characteristics (density, shape, etc.) by a variety of
methods or directly measured. One option is to calculate it from
sediment grain size following Dietrich (1982), using a Corey Shape
Factor (Corey, 1949) of 0.7 and a Powers roughness value (Powers,
1953) of 3.5.

Bed roughness is either specified or calculated using a variety of
methods. One option is to specify a Manning’s \( n \) and calculate the
roughness using the equations of MacWilliams (2004) that apply
Strickler (1923) formulae relating Manning’s \( n \) and depth.

\[ z_o = \exp \left( \frac{1}{n^2} \text{Lambert} W \left( \frac{ng + k_\text{H} \sqrt{H} \sqrt{g}}{H} \exp \left( -\frac{ng - \text{rgh} H + k_\text{H} \sqrt{H} \sqrt{g}}{ng} \right) \right) \right) \]  

(A6)

Where, \( n \) is Manning’s \( n \), \( g \) is the acceleration due to gravity, \( k \)
is von Karman’s constant (0.41), \( H \) is flow depth, \( I \) is a unit conversion
factor with units \( \text{ft}^{-1} \), and \( \text{Lambert W} \) solves the equation \( (w \exp w = x) \) for \( w \) as a function of \( x \).

A second option is specify a bottom roughness. A third option, the
one used by Jaffe and Gelfenbaum (2007) is to use the approach of
Wiberg and Rubin (1989) where bed roughness, \( z_{obd} \) is the com-
bination of a Nikuradse bed roughness (Nikuradse, 1933), \( z_{oBD} \) and
a roughness created by saltating sediment, \( z_{obr} \).

\[ z_{oBD} = z_{obd} + z_{obr} \]  

(A7)

The Nikuradse bed roughness is

\[ z_{obr} = \frac{\bar{d}}{30} \]  

(A8)

where \( \bar{d} \) is the mean grain diameter. The roughness from saltating
sediment is

\[ z_{obr} = \alpha_{BSD} \]  

(A9)

where \( \alpha_{BSD} \) is a constant equal to 0.056 and \( \alpha_{BSD} \) is the average saltation
height, which is a function of boundary and critical shear stresses,
using the formula from Wiberg and Rubin (1989).

It should be noted that the third option appears to underestimate
bed roughness during tsunamis where the bed is either not flat or
vegetation is present.

Eddy viscosity, \( K \), is given by Gelfenbaum and Smith (1986) as

\[ K = k U_e (g \tau^{1/2})^2 \]  

(A10)

where \( k \) is von Karmann’s constant, 0.41, and \( h \) is flow depth. This
formulation was developed as a best fit to laboratory data collected
under steady uniform flow conditions. Although much of a tsunami,
especially near the front, is anything but steady and uniform, portions
of a tsunami traveling on a gently sloping coastal plain may be quasi-
steady and quasi-uniform (Jaffe and Gelfenbaum, 2007).

The model iteratively adjusts sediment source distribution and shear
velocity (a parameterization of turbulent mixing intensity) to match the
observed bulk grain size distribution and thickness of the tsunami
deposit. For the first calculations, a guess of the shear velocity that
created the deposit and the tsunami deposit bulk grain size distribution
are used in Eqs. (3) through (10). For each size class, the reference
concentration, suspended sediment profile, and total suspended load
are calculated. The thickness and grain size distribution of the deposit
created from this sediment settling from suspension is compared to the
observed deposit. If the modeled deposit does not match the observed
deposit with the desired accuracy, the shear velocity and bed grain
size distribution are adjusted to create the observed deposit. Shear
velocity is increased (decreased) if the modeled deposit is too thin
(thick). Likewise, the fraction in each bed grain size class is increased
decreased) if the fraction in suspension is smaller (greater) than that in
the observed deposit. Adjustment of shear stress and grain size classes
is alternated. Total bottom roughness is also adjusted during each
iteration. The model is run until both the sediment in suspension for
each size class matches the fraction observed in the deposit and the deposit thickness matches the observed thickness to within 1%. If the initial guess for shear velocity does not result in model convergence, a new guess is made and the model is rerun. For runs on modern tsunami deposits, the model is not sensitive to the initial guess in shear velocity; however, if the deposit was not formed by sediment settling from suspension, the model does not converge because the size distribution in the deposit is not reproducible by this model.

References


